Pearson Edexcel Level 3

GCE Mathematics

Advanced

Paper 1: Pure Mathematics

PMT Mock 3

Time: 2 hours

Paper Reference(s)
9MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 16 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. The point $P(2,-3)$ lies on the curve with equation $y = \mathrm{f}(x)$.
State the coordinates of the image of P under the transformation represented by the curve

a. $y = |\mathrm{f}(x)|$

(1)

b. $y = \mathrm{f}(x-2)$

(1)

c. $y = 3\mathrm{f}(2x) + 2$

(2)

a. B1 (2,3) Accept without brackets. May be written $x = 2, y = 3$

b. B1 $(4,-3)$ Accept without brackets. May be written $x = 4, y = -3$

c. M1 either coordinate. e.g. $(1, \dots)$ or $(\dots, -7)$

A1 correct coordinates. $(1,-7)$

Accept without brackets. May be written $x = 1$ or $y = -7$

(Total for Question 1 is 4 marks)

2. $$f(x) = (2x - 3)(x - k) - 12$$

where k is a constant.

a. Write down the value of $f(k)$

(1)

When $f(x)$ is divided by $(x + 2)$ the remainder is -5

b. find the value of k.

(2)

c. Factorise $f(x)$ completely.

(3)

a. B1 for substituting $x = k$ and $f(k) = -12$
e.g. $f(k) = (2k - 3)(k - k) - 12 \Rightarrow f(k) = (2k - 3)(0) - 12 = -12$

b. M1 for substituting $x = -2$ and equating to –5 to form an equation in k and solving to find k
e.g. $f(-2) = (2(-2) - 3)(-2 - k) - 12 = -5$
$(-7)(-2 - k) - 12 = -5 \quad \Rightarrow -7(-2 - k) = 7$
$-2 - k = -1 \quad \Rightarrow k = \cdots.$

A1 $k = -1$

c. M1 for multiplying and substituting their constant value of k (in either order)
The multiplying-out may occur earlier.
e.g. $(2x - 3)(x + 1) - 12 = 2x^2 - x - 3 - 12 = 2x^2 - x - 15$

M1 for an attempt to factorise their three term quadratic
e.g. $2x^2 - x - 15 = (2x \pm 5)(x \pm 3)$

A1 The correct answer, as a product of factors, is required. e.g. $(2x + 5)(x - 3)$

(Total for Question 2 is 6 marks)

3. A circle C has equation

$$x^2 - 22x + y^2 + 10y + 46 = 0$$

a. Find

i. the coordinates of the centre A of the circle

ii. the radius of the circle.

(3)

Given that the points $Q(5,3)$ and S lie on C such that the distance QS is greatest,

b. find an equation of tangent to C at S, giving your answer in the form

$ax + by + c = 0$, where a, b and c are constants to be found.

(3)

a. i. M1 Attempts to complete the square on both x and y terms.

Accept $(x \pm 11)^2 + (y \pm 5)^2 = \cdots$ or imply this mark for a centre of $(\pm 11, \pm 5)$

e.g. $(x \pm 11)^2 - 11^2 + (y \pm 5)^2 - 5^2 + 46 = 0$

$(x \pm 11)^2 + (y \pm 5)^2 = 100$

A1 Correct centre $A(11, -5)$

Accept without brackets. May be written $x = 11, y = -5$

ii. A1 10

The M mark must have been awarded, so it can be scored following a centre of $(\pm 11, \pm 5)$

Do not allow for $\sqrt{100}$ or ± 10

b. B1 S is $(17, -13)$ or $m_T = \frac{3}{4}$

Either identifies the correct point S where A is the mid-point of QS or finds the correct gradient for the tangent using coordinates $(11, -5)$ and $(5,3)$ and takes negative reciprocal

e.g. $\left(\frac{5+x}{2}, \frac{3+y}{2}\right) = (11, -5) \Rightarrow \frac{5+x}{2} = 11$, $\frac{3+y}{2} = -5 \Rightarrow x = 17, y = -13$

or $m_{QA} = \frac{-5-3}{11-5} = -\frac{4}{3}$ and $m_T = \frac{3}{4}$

M1 for a full method to find the equation of a line e.g. attempts to find radius gradient and takes negative reciprocal and uses these to form the equation.

e.g $y + 13 = \frac{3}{4}(x - 17)$

A1 $3x - 4y - 103 = 0$ or any (non-zero) integer multiple of this. Accept terms in any order but have the $"= 0"$

(Total for Question 3 is 6 marks)

4. a. Express $\lim_{dx \to 0} \sum_{0.2}^{1.8} \frac{1}{2x}\, \delta x$ as an integral.

(1)

b. Hence show that

$$\lim_{dx \to 0} \sum_{0.2}^{1.8} \frac{1}{2x}\, \delta x = \ln k$$

where k is a constant to be found.

(2)

a. B1 States that $\int_{0.2}^{1.8} \frac{1}{2x}\ \mathrm{d}x$ or equivalent such as $\frac{1}{2}\int_{0.2}^{1.8} x^{-1}\ \mathrm{d}x$ but must include the limits and the dx.

b. M1 Know that $\int \frac{1}{x}\ \mathrm{d}x = \ln x$ and attempts to apply the limits (either way round)

Condone $\int \frac{1}{2x}\ \mathrm{d}x = p \ln x$ (including p =1) or $\int \frac{1}{2x}\ \mathrm{d}x = p \ln qx$ as long as the limits are applied.

Also be aware that $\int \frac{1}{2x}\ \mathrm{d}x = \ln x^{\frac{1}{2}}$, $\int \frac{1}{2x}\ \mathrm{d}x = \frac{1}{2}\ln|x| + c$ and $\int \frac{1}{2x}\ \mathrm{d}x = \frac{1}{2}\ln ax$ or equivalent are also correct.

$[p \ln x]_{0.2}^{1.8} = p \ln 1.8 - p \ln 0.2$ is sufficient evidence to award this mark

e.g $\int_{0.2}^{1.8} \frac{1}{2x}\ \mathrm{d}x = \left[\frac{1}{2}\ln|x|\right]_{0.2}^{1.8} = \frac{1}{2}[\ln 1.8 - \ln 0.2\,] = \frac{1}{2}\ln\frac{1.8}{0{,}2} = \frac{1}{2}\ln 9 = \ln 9^{\frac{1}{2}}$

A1 Correct solution only $\ln 3$

(Total for Question 4 is 3 marks)

5. A scientist is studying a population of lizards on an island and uses the linear model $P = a + bt$ to predict the future population of the lizard where P is the population and t is the time in years after the start of the study.

Given that

- The number of population was 900, exactly 5 years after the start of the study.
- The number of population was 1200, exactly 8 years after the start of the study.

a. find a complete equation for the model.

(4)

b. Sketch the graph of the population for the first 10 years.

(1)

c. Suggest one criticism of this model.

(1)

a. M1 For translating the problem into mathematics. Attempts to use the given equation or equivalent with either or the piece of information to form one correct equation.

e.g. $t = 5, P = 900 \Rightarrow a + 5b = 900$ or $t = 8, P = 1200 \Rightarrow a + 8b = 1200$

A1 Two correct equations e.g. $a + 5b = 900$ and $a + 8b = 1200$

M1 Solves simultaneously to find values for a and b

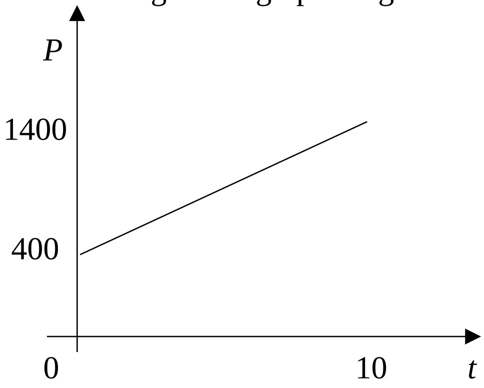
e.g. $a + 5b = 900$

$-a - 8b = -1200 \quad \Rightarrow \quad b = 100\,,\ a = 400$

A1 Establishes the full equation of the model with values of a and b.

e.g. $P = 400 + 100t$

b. B1 A straight line graph P against t with coordinates (0,400) and (10,1400)



c. B1 Population growth is usually modelled exponentially.

(Total for Question 5 is 6 marks)

6.

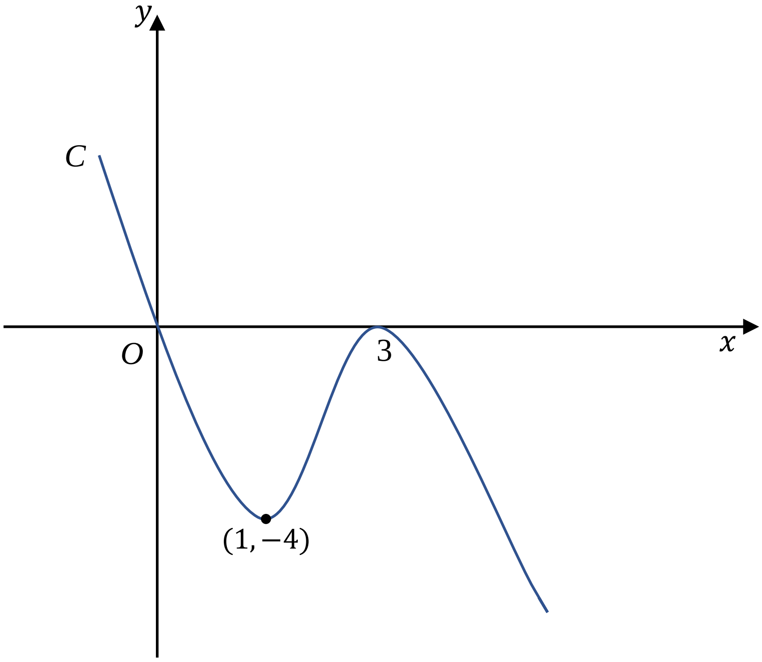


Figure 1

The figure 1 shows sketch of the curve C with equation $y = \mathrm{f}(x)$.

$$\mathrm{f}(x) = ax(x-b)^2, x \in R$$

where a and b are constants.

The curve passes through the origin and touches the x-axis at the point (3,0).

There is a minimum point at (1,–4) and a maximum point at (3,0).

a. Find the equation of C.

(3)

b. Deduce the values of x for which

$$\mathrm{f}'(x) > 0$$

(1)

Given that the line with equation $y = k$, where k is a constant, intersects C at exactly one point,

c. State the possible values for k.

(2)

a. M1 Realises that the equation of C is of the form $y = ax(x-3)^2$. Condone with $a = 1$ for this mark.

dM1 Substitutes (1,–4) into the form $y = ax(x-3)^2$ and attempts to find the value for a.

e.g. $-4 = a(1)\left((1)-3\right)^2 \Rightarrow -4 = 4a \Rightarrow a = \cdots$

A1 Uses all of the information to form a correct equation for C.

e.g. $a = -1 \Rightarrow y = -x(x-3)^2$

OR

M1 Realises that the equation of C is of the form $y = ax^3 + bx^2 + cx$ and forms two equations in a, b and c. Condone with $a = 1$ for this mark

There are four equations that could be formed, only two are necessary for this mark.

Using $(1,-4)$ $-4 = a + b + c$

Using $(3,0)$ $0 = 27a + 9b + 3c \Rightarrow 0 = 9a + 3b + c$

Using $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ at $x = 1$ $3ax^2 + 2bx + c = 0 \Rightarrow 3a + 2b + c = 0$

Using $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ at $x = 3$ $3ax^2 + 2bx + c = 0 \Rightarrow 27a + 6b + c = 0$

dM1 Forms and solves three different equations, one of which must be using

$(1,-4)$ to find values for a, b and c.

e.g. $a + b + c = -4$ (1)

$9a + 3b + c = 0$ (2) subtract (1) from (2) $\Rightarrow 8a + 2b = 4$

$9a + 3b + c = 0$ (2)

$3a + 2b + c = 0$ (3) subtract (3) from (2) $\Rightarrow 6a + b = 0$

Solves $8a + 2b = 4$ and $6a + b = 0$ simultaneously $\Rightarrow a = -1, b = 6, c = -9$

A1 Uses all of the information to form a correct equation for C.

$y = -x^3 + 6x^2 - 9x = -x(x^2 - 6x + 9) = -x(x-3)^2$

b. B1 Deduces $1 < x < 3$ or equivalent such as $x > 1, x < 3$ $x > 1$ and $x < 3$

$\{x: x > 1\} \cap \{x: x < 3\}$ $x \in (1,3)$

c. M1 States either $k > 0$ or $k < -4$

A1 Fully correct solution in the form $\{k: k > 0\} \cup \{k: k < -4\}$

(Total for Question 6 is 6 marks)

7. (i) Given that a and b are integers such that

$$a + b \text{ is odd}$$

Use algebra to prove by contradiction that at least one of a and b is odd.

(3)

(ii) A student is trying to prove that

$$(p+q)^2 < 13p^2 + q^2 \qquad \text{where } p < 0$$

The student writes:

$$p^2 + 2pq + q^2 < 13p^2 + q^2$$
$$2pq < 12p^2$$
$$\text{so as } p < 0 \qquad 2q < 12p$$
$$q < 6p$$

a. Identify the error made in the proof.

(1)

b. Write out the correct solution.

(1)

i. B1 For using the "correct" /allowable language in setting up the contradiction. Expect to see a minimum of

- "assume" or "let" or "there is" or other similar words
- "$a + b$ is odd" and "neither a nor b is odd"

"There exists integers a and b such that $a + b$ is odd then neither a nor b is odd"

M1 Sets $a = 2k$ and $b = 2m$ and then attempts $a + b = 2k + 2m = \cdots$

A1 Obtains $a + b = 2k + 2m = 2(k + m)$

States that $a + b$ is even, giving a contradiction that $a + b$ is odd.

"if $a + b$ is odd that at least one of a and b is odd"

ii. a. B1 Identifies the error and states that as $p < 0 \Rightarrow \quad 2q > 12p$

b. B1 Correct solution only $\quad q > 6p$

(Total for Question 7 is 5 marks)

8.

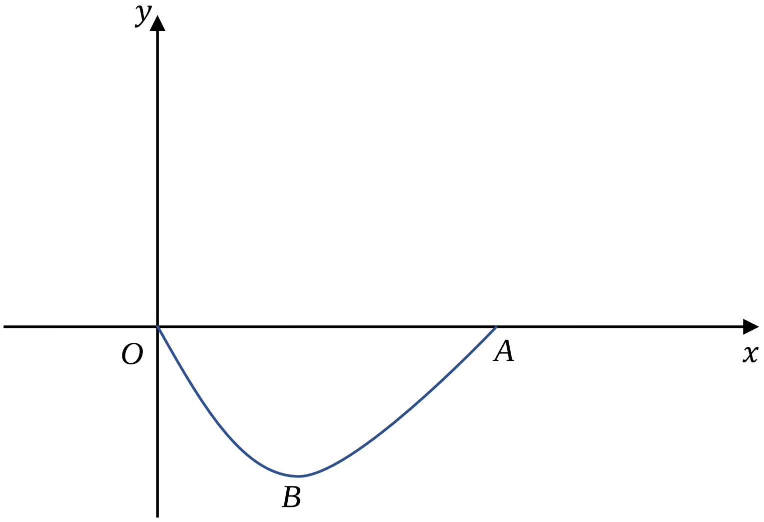


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where $x \in R$, $x > 0$

$$\mathrm{f}(x) = (0.5x - 8)\ln(x+1) \qquad 0 \leq x \leq A$$

a. Find the value of A.

(1)

b. Find $\mathrm{f}'(x)$

(2)

The curve has a minimum turning point at B.

c. Show that the x-coordinate of B is a solution of the equation

$$x = \frac{17}{\ln(x+1)+1} - 1$$

(2)

d. Use the iteration formula

$$x_{n+1} = \frac{17}{\ln(x_n+1)+1} - 1$$

with $x_0 = 5$ to find the values of x_1 and the value of x_6 giving your answers to three decimal places.

(3)

a. B1 states that $A = 16$ solving $0.5x - 8 = 0 \quad \Rightarrow x = \frac{8}{0.5} = 16$

b. M1 Attempts to differentiate using the product rule

Look for $(0.5x - 8) \times \frac{1}{(x+1)} \pm k\ln(x+1)$, where k is a constant,

You will see attempts from $f(x) = 0.5x\ln(x+1) - 8\ln(x+1)$ which can be similarly marked.

In this case look for $\pm\frac{0.5x}{(x+1)} \pm b\ln(x+1) - \frac{c}{(x+1)}$

A1 Correct differentiation

$\frac{dy}{dx} = (0.5x - 8) \times \frac{1}{(x+1)} + 0.5\ln(x+1)$, or equivalent such as

$\frac{dy}{dx} = \frac{0.5x}{(x+1)} + 0.5\ln(x+1) - \frac{8}{(x+1)}$

c. M1 Score for setting their $\frac{dy}{dx} = 0$ and proceeding to an equation where the variable x occurs only once.

e.g. $(0.5x - 8) \times \frac{1}{(x+1)} + 0.5\ln(x+1) = 0$

$$0.5\ln(x+1) = (8 - 0.5x) \times \frac{1}{(x+1)}$$

$$\ln(x+1) = \frac{16-x}{(x+1)} \qquad \text{(Dividing } (16-x) \text{ by } (x+1)\text{)}$$

$$\begin{array}{rl} & \underline{-1} \\ x+1) & \overline{-x+16} \\ & \underline{-x-1} \\ & 17 \end{array}$$

$$\ln(x+1) = -1 + \frac{17}{(x+1)}$$

Or e.g.

$$\frac{0.5x}{(x+1)} + 0.5\ln(x+1) - \frac{8}{(x+1)} = 0$$
$$0.5\ln(x+1) = (8 - 0.5x) \times \frac{1}{(x+1)}$$
$$0.5(x+1)\ln(x+1) = 8 - 0.5x$$
$$0.5x\ln(x+1) + 0.5\ln(x+1) = 8 - 0.5x$$
$$0.5x\ln(x+1) + 0.5x = 8 - 0.5\ln(x+1)$$
$$0.5x(\ln(x+1) + 1) = 8 - 0.5\ln(x+1)$$

A1 correctly proceeds to the given answer of $x = \frac{17}{\ln(x+1)+1} - 1$ showing all key steps.

e.g.

$$\ln(x+1)+1 = \frac{17}{(x+1)}$$
$$(x+1)(\ln(x+1)+1) = 17$$
$$x+1 = \frac{17}{\ln(x+1)+1}$$
$$x = \frac{17}{\ln(x+1)+1} - 1$$

Or e.g.

$$x(\ln(x+1)+1) = 16 - \ln(x+1)$$

$x = \frac{16-\ln(x+1)}{\ln(x+1)+1}$ dividing $16 - \ln(x+1)$ by $\ln(x+1)+1$

$$\begin{array}{r} -1 \\ \ln(x+1)+1\,\overline{\smash{)}-\ln(x+1)+16} \\ -\ln(x+1)-1 \\ \hline 17 \end{array}$$

$$x = \frac{17}{\ln(x+1)+1} - 1$$

d. M1 Attempts to use the iteration formula at least once.

Usually to find $x_1 = \frac{17}{\ln(5+1)+1} - 1$ which may be implied by awrt 5.089

A1 $x_1 =$ awrt 5.089

A1 $x_6 = 5.066$

(Total for Question 8 is 8 marks)

9.

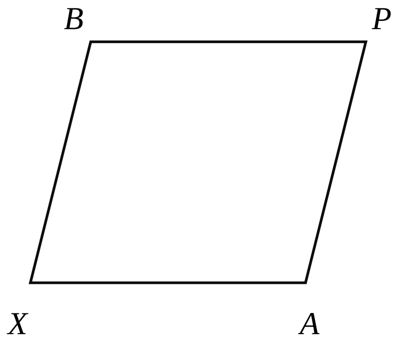


Figure 3

Figure 3 shows a sketch of a parallelogram *XAPB*.

Given that $\overrightarrow{OX} = \begin{pmatrix}1\\2\\3\end{pmatrix}$

$\overrightarrow{OA} = \begin{pmatrix}0\\4\\1\end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix}3\\3\\1\end{pmatrix}$

a. Find the coordinates of the point *P*.

(3)

b. Show that *XAPB* is a rhombus.

(2)

c. Find the exact area of the rhombus *XAPB*.

(3)

a. M1 For attempting one of $\overrightarrow{XB}$ or $\overrightarrow{BX}$ or $\overrightarrow{XA}$ or $\overrightarrow{AX}$. It must be correct for at least one of the components.

$$\text{e.g. } \overrightarrow{XB} = \overrightarrow{OB} - \overrightarrow{OX} = \begin{pmatrix}3\\3\\1\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}2\\1\\-2\end{pmatrix}$$

$$\text{or } \overrightarrow{XA} = \overrightarrow{OA} - \overrightarrow{OX} = \begin{pmatrix}0\\4\\1\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\2\\-2\end{pmatrix}$$ either way round

M1 For attempting $\overrightarrow{OP} = \overrightarrow{OB} - \overrightarrow{OX} + \overrightarrow{OA}$. It must be correct for at least one of the components.

$$\text{e.g. } \overrightarrow{OP} = \overrightarrow{OB} - \overrightarrow{OX} + \overrightarrow{OA} = \begin{pmatrix}3\\3\\1\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} + \begin{pmatrix}0\\4\\1\end{pmatrix} = \begin{pmatrix}2\\5\\-1\end{pmatrix}$$

$$\text{or } \overrightarrow{OP} = \overrightarrow{OA} - \overrightarrow{OX} + \overrightarrow{OB} = \begin{pmatrix}0\\4\\1\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} + \begin{pmatrix}3\\3\\1\end{pmatrix} = \begin{pmatrix}2\\5\\-1\end{pmatrix}$$

A1 Correct answer only. $\overrightarrow{OP} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ or $\overrightarrow{OP} = \begin{pmatrix}2\\5\\-1\end{pmatrix}$

b. M1 Attempts both $|\overrightarrow{XB}| = \sqrt{2^2 + 1^2 + (\pm 2)^2}$ and $|\overrightarrow{XA}| = \sqrt{(\pm 1)^2 + 2^2 + (\pm 2)^2}$

If $\overrightarrow{XA}$ or $\overrightarrow{XB}$ has not been found in part a, it need to be calculated in part b.

Alternatively attempts $\overrightarrow{XP} \bullet \overrightarrow{BA}$ or XM^2, MB^2 and XB^2 where M is the mid point of XP

A1 Shows that $\overrightarrow{XB} = \overrightarrow{XA} = 3$ and states *XAPB* is a rhombus.

Requires both a reason and a conclusion.

In the **alternatives** $\overrightarrow{XP} \bullet \overrightarrow{BA} = (\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) \bullet (-3\mathbf{i} + \mathbf{j}) = -3 + 3 + 0 = 0$ so diagonals cross at 90^0 so *XAPB* is a rhombus

or $XM^2 + MB^2 = XB^2 = 6.5 + 2.5 = 9 \Rightarrow \angle XMB = 90^0 \Rightarrow$ rhombus

c. M1 Attempts to find both $\overrightarrow{XP} = \overrightarrow{XB} + \overrightarrow{XA} = (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + (-\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

$$\overrightarrow{XP} = (\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$$

and $\overrightarrow{BA} = \overrightarrow{BX} + \overrightarrow{XA} = (-2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + (-\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) = (-3\mathbf{i} + \mathbf{j})$

You may see $\overrightarrow{XM} = \frac{1}{2}\overrightarrow{XB} + \frac{1}{2}\overrightarrow{BP} = \frac{1}{2}(2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + \frac{1}{2}(-\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

$$\overrightarrow{XM} = (\tfrac{1}{2}\mathbf{i} + \tfrac{3}{2}\mathbf{j} - 2\mathbf{k})$$

and $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BX} + \frac{1}{2}\overrightarrow{XA} = \frac{1}{2}(-2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \frac{1}{2}(-\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) = \left(-\frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}\right)$

M1 Attempts to find the area *XAPB*.

e.g. Area $= \frac{1}{2}\,|\overrightarrow{XP}| \times |\overrightarrow{BA}| = \frac{1}{2} \times \sqrt{1^2 + 3^2 + (\pm 4)^2} \times \sqrt{(\pm 3)^2 + 1^2} = \frac{1}{2}\sqrt{26} \times \sqrt{10}$

Alternatively the sum of the area of four right angled triangles.

e.g. Area = $4 \times \frac{1}{2} \times |\overrightarrow{XM}| \times |\overrightarrow{BM}| = 4 \times \frac{1}{2} \times \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + (\pm 2)^2} \times \sqrt{\left(\pm\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$$\text{Area} = 4 \times \frac{1}{2}\sqrt{\frac{13}{2}} \times \sqrt{\frac{5}{2}}$$

A1 correct answer only $\sqrt{65}$

Alternatively

M1 Attempts to find the angle $\cos XBP$ or $\cos BPA$ using the formula

$$\overrightarrow{BX}.\overrightarrow{BP} = \frac{1}{2}\,|\overrightarrow{BX}| \times |\overrightarrow{BP}| \cos XBP$$

e.g $\pm\begin{pmatrix}-2\\-1\\2\end{pmatrix}.\pm\begin{pmatrix}-1\\2\\-2\end{pmatrix} = \sqrt{(\pm 2)^2 + (\pm 1)^2 + (2)^2} \times \sqrt{(\pm 1)^2 + 2^2 + (\pm 2)^2} \cos XBP$

$$2 - 2 - 4 = 3 \times 3 \times \cos XBP \quad \Rightarrow \quad \cos XBP = -\frac{4}{9}$$

M1 Constructs a rigorous method leading to the area $XAPB$

e.g Area $XAPB$ = $2 \times \frac{1}{2} \times$ Area of triangle $XBP = 2 \times \frac{1}{2} \times \sqrt{9} \times \sqrt{9} \sin XBP$

where $\sin XBP = \sqrt{1 - \cos^2 XBP} = \sqrt{1 - \left(\frac{-4}{9}\right)^2} = \frac{\sqrt{65}}{9}$

A1 correct answer only $\sqrt{65}$

(Total for Question 9 is 8 marks)

10. The figure 4 shows the curves $\mathrm{f}(x) = A - Be^{-0.5x}$ and $\mathrm{g}(x) = 26 + e^{0.5x}$

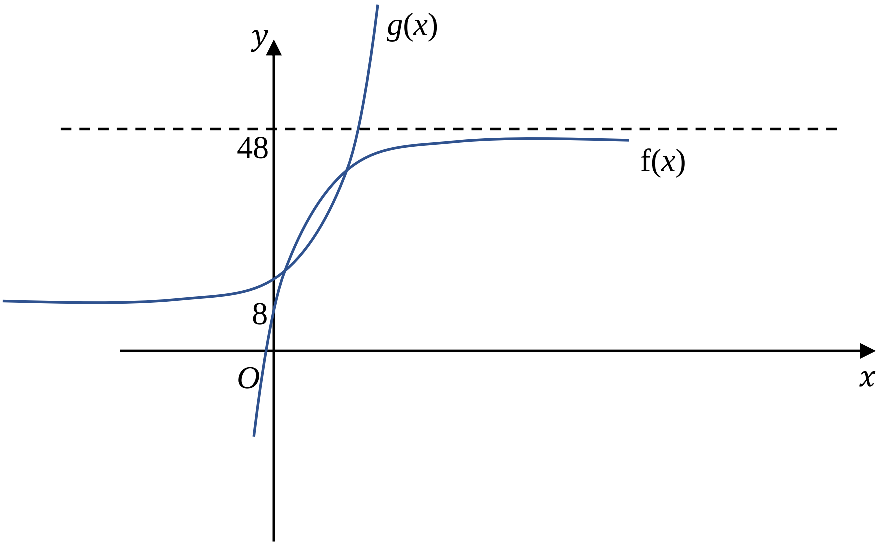


Figure 4

Given that $\mathrm{f}(x)$ passes through (0,8) and has an horizontal asymptote $y = 48$

a. Find the values of A and B for $\mathrm{f}(x)$

(3)

b. State the range of $\mathrm{g}(x)$

(1)

The curves $\mathrm{f}(x)$and $\mathrm{g}(x)$ meet at the points C and D

c. Find the x-coordinates of the intersection points C and D, in the form $\ln k$, where k is an integer.

(4)

a. B1 $A = 48$

M1 Substitutes $x = 0$ and $y = 8$ into $f(x)$ and attempts to find the value of B

e.g. $8 = 48 - Be^{-0.5(0)} \Rightarrow 8 = 48 - B \Rightarrow B = \cdots$

A1 $B = 40$

b. B1 Correct range $g(x) > 26$. Allow equivalent notation. e.g. $y > 26, g > 26,$ $y \in (26, \infty)$

c. M1 Sets their "$48 - 40$"$e^{-0.5x} = 26 + e^{0.5x}$ and rearranges to produce a simplified equation of the form $e^{0.5x} + 40e^{-0.5x} - 22 = 0$

e.g. $48 - 40"e^{-0.5x} = 26 + e^{0.5x} \Rightarrow e^{0.5x} + 40e^{-0.5x} - 22 = 0$

A1 Correct quadratic equation.

Look for $(e^{0.5x})^2 - 22e^{0.5x} + 40 = 0$ or $e^x - 22e^{0.5x} + 40 = 0$

e.g. $e^{0.5x} + 40e^{-0.5x} - 22 = 0 \Rightarrow e^{0.5x} + \frac{40}{e^{0.5x}} - 22 = 0$ multiply each term by $e^{0.5x}$

$$\Rightarrow (e^{0.5x})^2 - 22e^{0.5x} + 40 = 0$$

M1 Full attempt to find the value of x.

This involves solving a 3TQ in $e^{0.5x}$ followed by the use of lns.

You may see different variables such as t

e.g. $t = e^{0.5x}$, $t^2 - 22t + 40 = 0$, $(t-20)(t-2) = 0 \Rightarrow t = 20, t = 2$

$\Rightarrow e^{0.5x} = 20 \Rightarrow x = 2\ln 20$, $e^{0.5x} = 2 \Rightarrow x = 2\ln 2$

A1 Correct answers only e.g. $\ln 400$, $\ln 4$

(Total for Question 10 is 8 marks)

11.

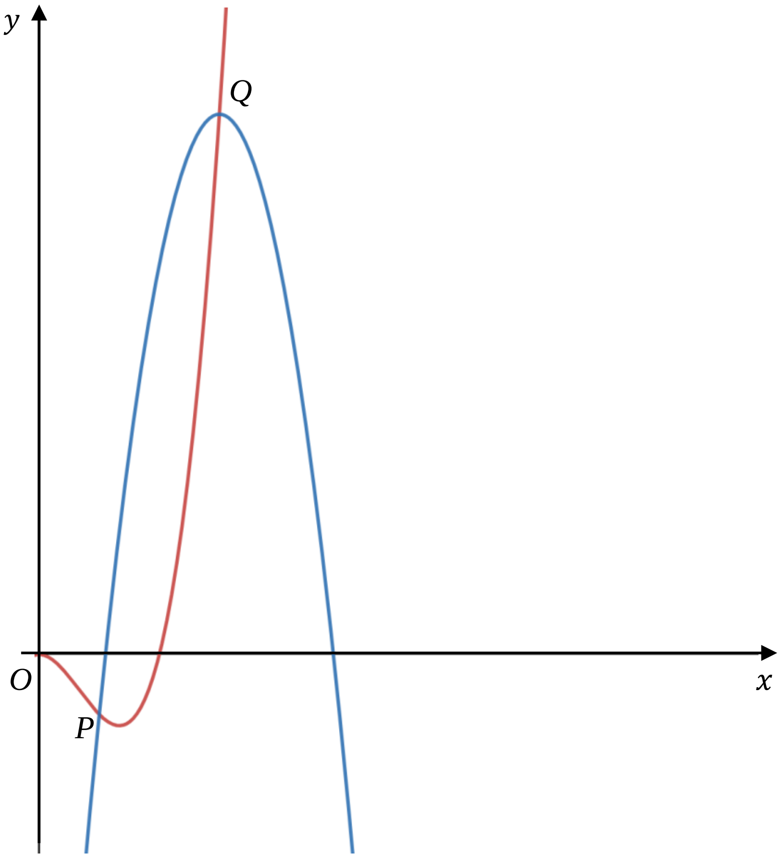


Figure 5

The figure 5 shows part of the curves C_1 and C_2 with equations

$$C_1: y = x^3 - 2x^2 \qquad x > 0$$

$$C_2: y = 9 - \frac{5}{2}(x-3)^2 \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q.

a. Verify that the point Q has coordinates (3,9)

(1)

b. Use algebra to find the coordinates of the point P.

(6)

a. B1 Substitutes $x = 3$ into both $y = x^3 - 2x^2$ and $y = 9 - \frac{5}{2}(x-3)^2$ and achieves $y = 9$ in both.

e.g. $y = (3)^3 - 2(3)^2 = 9$ and $y = 9 - \frac{5}{2}(3-3)^2 = 9$

b. B1 Sets equations equal to each other and proceeds to $2x^3 + x^2 - 30x + 27 = 0$

e.g. $x^3 - 2x^2 = 9 - \frac{5}{2}(x-3)^2 \Rightarrow 2x^3 - 4x^2 = 18 - 5x^2 + 30x - 45$

M1 Divides by $(x-3)$ to form a quadratic factor. Allow any suitable algebraic method including division or inspection.

If attempted via inspection look for correct first term and last terms

e.g. $2x^3 + x^2 - 30x + 27 = (x-3)(2x^2 + ax \pm 9)$ if cubic expression is correct

If attempted via division look for correct first and second terms

e.g.

$$\begin{array}{r} 2x^2+7x \\ x-3\overline{)2x^3+x^2-30x+27} \end{array}$$

if cubic expression is correct

A1 $2x^3 + x^2 - 30x + 27 = (x-3)(2x^2 + 7x - 9)$

M1 Solves their quadratic equation $2x^2 + 7x - 9 = 0$ using a suitable method including calculator.

e.g. $2x^2 + 7x - 9 = (2x+9)(x-1) = 0 \Rightarrow x = -4.5\,, x = 1$

A1 Gives $x = 1$ only

A1 Coordinates of $P = (1,-1)$

(Total for Question 11 is 7 marks)

12.

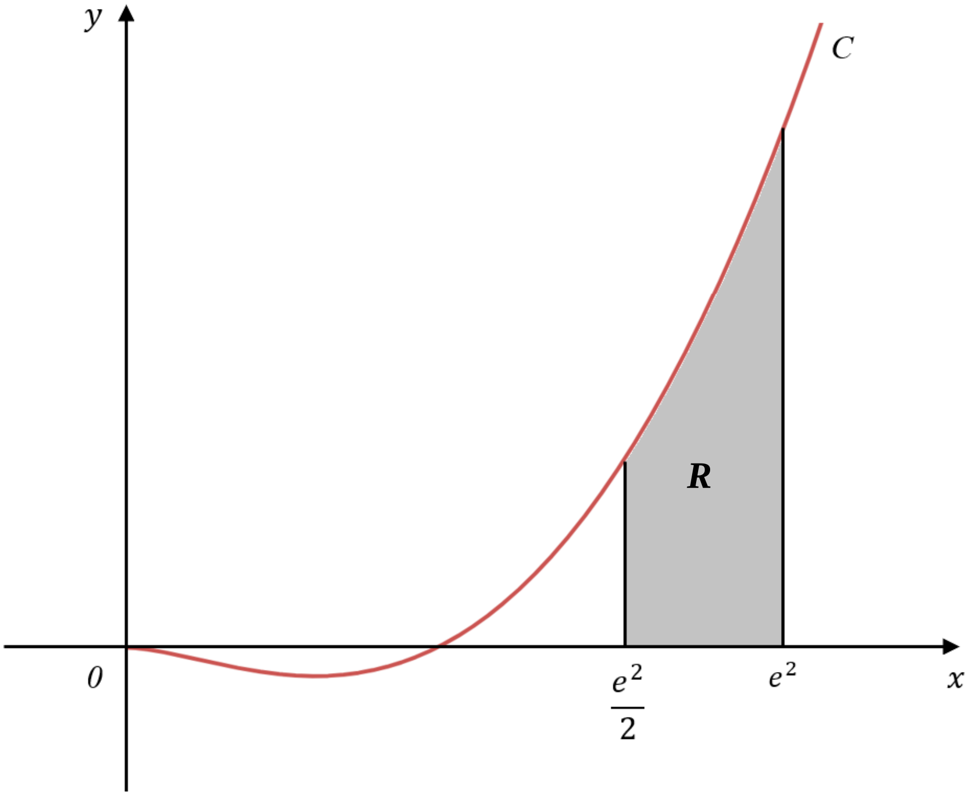


Figure 6

The figure 6 shows a sketch of the curve with equation

$$y = x^2 \ln 2x$$

The finite region R, shown shaded in figure 5, is bounded by the line with equation $x = \frac{e^2}{2}$, the curve C, the line with equation $x = e^2$ and the x-axis.

Show that the exact value of the area of region R is $\frac{e^6}{72}(35 + 24 \ln 2)$.

M1 Attempts by parts to reach a form $\int x^2 \ln 2x \ \ \mathrm{d}x = \pm ax^3 \ln 2x \pm b \int x^3 \times \frac{1}{x} \,\mathrm{d}x$

where $a, b \neq 0$

If a formula is stated it must be correct.

e.g. $u = \ln 2x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = x^2 \quad \Rightarrow v = \frac{x^3}{3}$

$$\int x^2 \ln 2x \ \ \mathrm{d}x = \pm ax^3 \ln 2x \pm b \int x^3 \times \frac{1}{x} \,\mathrm{d}x$$

dM1 Integrates $\int x^3 \times \frac{1}{x} \,\mathrm{d}x$ to reach a form $\pm bx^3$

A1 correct answer only $\int x^2 \ln 2x \ \ \mathrm{d}x = \frac{x^3}{3} \ln 2x - \frac{x^3}{9}$

M1 Uses correct limits correct way round in an integrated function to find the area of the region and attempts to simplify by using one log law correctly.

e.g. $$\left[\frac{x^3}{3}\ln 2x - \frac{x^3}{9}\right]_{\frac{e^2}{2}}^{e^2} = \left(\frac{(e^2)^3}{3}\ln 2(e^2) - \frac{(e^2)^3}{9}\right) - \left(\frac{\left(\frac{e^2}{2}\right)^3}{3}\ln 2\left(\frac{e^2}{2}\right) - \frac{\left(\frac{e^2}{2}\right)^3}{9}\right)$$

$$= \left(\frac{e^6}{3}(\ln 2 + \ln e^2) - \frac{e^6}{9}\right) - \left(\frac{\left(\frac{e^6}{8}\right)}{3}\ln e^2 - \frac{\left(\frac{e^6}{8}\right)}{9}\right)$$

$$= \left(\frac{e^6}{3}\ln 2 + \frac{2e^6}{3}\ln e - \frac{e^6}{9}\right) - \left(\frac{2e^6}{24}\ln e - \frac{e^6}{72}\right)$$

$$= \frac{24e^6}{72}\ln 2 + \frac{35e^6}{72}$$

A1 Correct answer only. $\frac{e^6}{72}(35 + 24\ln 2)$.

(Total for Question 12 is 5 marks)

13. A construction company had a 30-year programme to build new houses in Newtown. They began in the year 1991 (Year 1) and finished in 2020 (Year 30).
The company built 120 houses in year 1, 140 in year 2, 160 houses in year 3 and so on, so that the number of houses they built form an arithmetic sequence.
A total of 8400 new houses were built in year *n*.

a. Show that

$$n^2 + 11n - 840 = 0$$

(2)

b. Solve the equation

$$n^2 + 11n - 840 = 0$$

and hence find in which year 8400 new houses were built.

(2)

a. M1 Uses the information given to set a correct equation in *n*.
The values of *S*, *a* and *d* need to be correct and used within a correct formula
Possible ways to score this include unsimplified versions
e.g. $S = 8400$, $a = 120$ and $d = 20$ $\Rightarrow$ $8400 = \frac{n}{2}(2 \times 120 + (n-1) \times 20)$

A1 Proceeds without error to the given answer. Look at least a line with the brackets correctly removed as well as a line with the terms in *n* correctly combined.
e.g. $8400 = \frac{n}{2}(2 \times 120 + (n-1) \times 20) \Rightarrow 8400 = n(120 + 10n - 10)$

$$8400 = 120n + 10n^2 - 10n \quad \Rightarrow \quad 10n^2 + 110n - 8400 = 0$$

$$n^2 + 11n - 840 = 0$$

b. B1 $n = 24, -35$
B1 Chooses $n = 24$ and finds Year 2014

(Total for Question 13 is 4 marks)

14. Given that

$$2\cos(x+60)^0 = \sin(x-30)^0$$

a. Show, without using a calculator, that

$$\tan x = \frac{\sqrt{3}}{3}$$

(4)

b. Hence solve, for $0 \leq \theta < 360^0$

$$2\cos(2\theta+90)^0 = \sin(2\theta)^0$$

(4)

a. M1 Attempts to use both compound angle expressions to set up an equation in $\sin x$ and $\cos x$

Condone missing bracket and incorrect signs but the terms must be correct

e.g. $\cos(x+60)^0 = \pm\cos x\cos 60 \pm \sin x \sin 60$

$\sin(x-30)^0 = \pm\sin x\cos 30 \pm \cos x\sin 30$

A1 correct equation $2\cos x\cos 60 - 2\sin x\sin 60 = \sin x\cos 30 - \cos x \sin 30$

M1 Shows the necessary progress towards showing the given result.

There are three key moves, two of which must be shown for this mark.

- Uses $\frac{\sin x}{\cos x} = \tan x$ to form an equation in just $\tan x$
- Uses exact numerical values for $\sin 30^0, \sin 60^0, \cos 30^0, \cos 60^0$ with at least two correct
- Collect terms in $\sin x$ and $\cos x$ or alternatively in $\tan x$

e.g. $2\cos x \times \frac{1}{2} - 2\sin x \times \frac{\sqrt{3}}{2} = \sin x \times \frac{\sqrt{3}}{2} - \cos x \times \frac{1}{2}$

$\Rightarrow\ 2\cos x - 2\sqrt{3}\sin x = \sqrt{3}\sin x - \cos x$

$\Rightarrow\ 3\cos x = 3\sqrt{3}\sin x$ or $\frac{3}{3\sqrt{3}} = \tan x$

A1 Proceeds to the given answer with accurate work showing all necessary lines.

e.g $\tan x = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

b. B1 Deduces that $x = 2\theta + 30$ or equivalent such as $\theta = \frac{x-30^0}{2}$

This is implied for sight of the equation $\tan(2\theta + 30) = \frac{\sqrt{3}}{3}$

M1 Proceeds from $\tan(2\theta \pm \alpha^0) = \frac{\sqrt{3}}{3} \Rightarrow 2\theta \pm \alpha^0 = 30^0, 210^0, 390^0, 570^0, \ldots$

where $\alpha \neq 0$

One angle for $\arctan(\frac{\sqrt{3}}{3})$ must be correct in degrees or radians. Radians answers $\frac{\pi}{6}, \frac{7\pi}{6}, \ldots$

dM1 Attempts at the correct method to find one value of θ from their

$2\theta \pm \alpha^0 = 30^0$ to $\theta = \frac{30^0 \pm \alpha}{2}$

e.g. $\tan(2\theta + 30^0) = \frac{\sqrt{3}}{3} \Rightarrow \theta = 0^0$

A1 $\theta = 0^0, 90^0, 180^0, 270^0$ with no other values given in the range

(Total for Question 14 is 8 marks)

15.

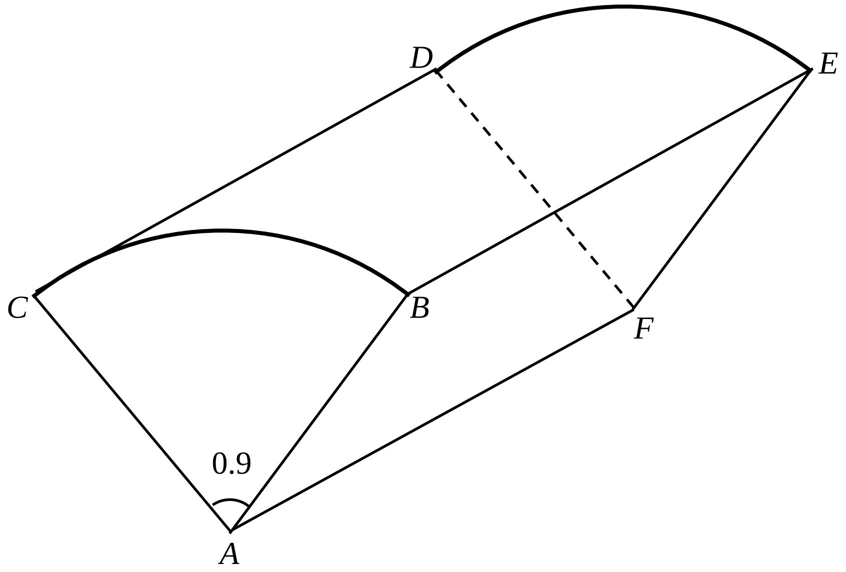


Figure 7

Figure 7 shows an open tank for storing water, *ABCDEF*. The sides *ACDF* and *ABEF* are rectangles. The faces *ABC* and *FED* are sectors of a circle with radius *AB* and *FE* respectively.

- $AB = FE = r$ cm
- $AF = BE = CD = l$ cm
- angle BAC = angle EFD = 0.9 radians

Given that the volume of the tank is 360 cm^3

a. show that the surface area of the tank, S cm^2, is given by

$$S = 0.9r^2 + \frac{1600}{r}$$

(4)

Given that r can vary

b. use calculus to find the value of r for which S is stationary.

(4)

c. Find, to 3 significant figures the minimum value of S.

(2)

a. M1 Attempts to use the fact that the volume of the tank is 360 cm^3

Sight of $\frac{1}{2}r^2 \times 0.9 \times l = 360$ leading to $l = \cdots$ or $rl = \cdots$ scores this mark

But condone an equation of the correct form so allow for $kr^2 l = 360 \Rightarrow l = \cdots$ or $rl = \cdots$

e.g. $\frac{1}{2}r^2 \times 0.9 \times l = 360 \quad \Rightarrow \quad l = \frac{2\times360}{0.9r^2}$

A1 A correct expression for $l = \frac{720}{0{,}9r^2}$ or $rl = \frac{800}{r}$ which may left unsimplified

This may be implied by an expression for S or part of S

e.g. $2rl = 2r \times \frac{800}{r^2}$

dM1 Attempts to substitute their $l = \frac{a}{r^2}$ or equivalent such as $lr = \frac{a}{r}$ into a correct expression for S

Sight of $\frac{1}{2}r^2 \times 0.9 + \frac{1}{2}r^2 \times 0.9 + rl + rl$ with an appropriate substitution

Simplified versions such as $0.9r^2 + 2rl$ used with an appropriate substitution is fine.

e.g. $S = \frac{1}{2}r^2 \times 0.9 + \frac{1}{2}r^2 \times 0.9 + rl + rl = \frac{1}{2}r^2 \times 0.9 + \frac{1}{2}r^2 \times 0.9 + r \times \frac{800}{r^2} + r \times \frac{800}{r^2}$

or $S = 0.9r^2 + 2rl = 0.9r^2 + 2r \times \frac{800}{r^2}$

A1 Correct work leading to the given result.

$S = \cdots, SA = \cdots$ or surface area must be seen at least once in the correct place.

The method must be made clear so expect to see evidence.

e.g. $S = 0.9r^2 + 2rl \Rightarrow 0.9r^2 + 2r \times \frac{720}{0.9r^2} \Rightarrow 0.9r^2 + 2r \times \frac{800}{r^2} \Rightarrow 0.9r^2 + \frac{1600}{r}$

b. M1 achieves a derivative of the form $ar \pm \frac{b}{r^2}$ where a and b are non-zero constants.

A1 Achieves $\left(\frac{\mathrm{d}S}{\mathrm{d}r}\right) = 1.8r - \frac{1600}{r^2}$

dM1 Sets or implies that their $\frac{\mathrm{d}S}{\mathrm{d}r} = 0$ and proceeds to $pr^3 = q,\ p \times q > 0$. It is dependent upon a correct attempt at differentiation. This mark may be implied by a correct answer to their $ar - \frac{b}{r^2} = 0$

e.g $1.8r - \frac{1600}{r^2} = 0 \quad \Rightarrow 1.8r^3 - 1600 = 0 \quad \Rightarrow \quad 1.8r^3 = 1600$

A1 $r^3 = \frac{8000}{9} \quad \Rightarrow \quad r = \text{awrt } 9.61 \quad \text{or} \quad r = \sqrt[3]{\frac{8000}{9}}$

c. M1 Substituting found value of r into $S = 0.9r^2 + \frac{1600}{r}$

e.g. $S = 0.9(9.61499\,\ldots)^2 + \frac{1600}{9.61499\ldots}$

A1 $S = 249.61\,\ldots$ awrt 250

(Total for Question 15 is 10 marks)

16.

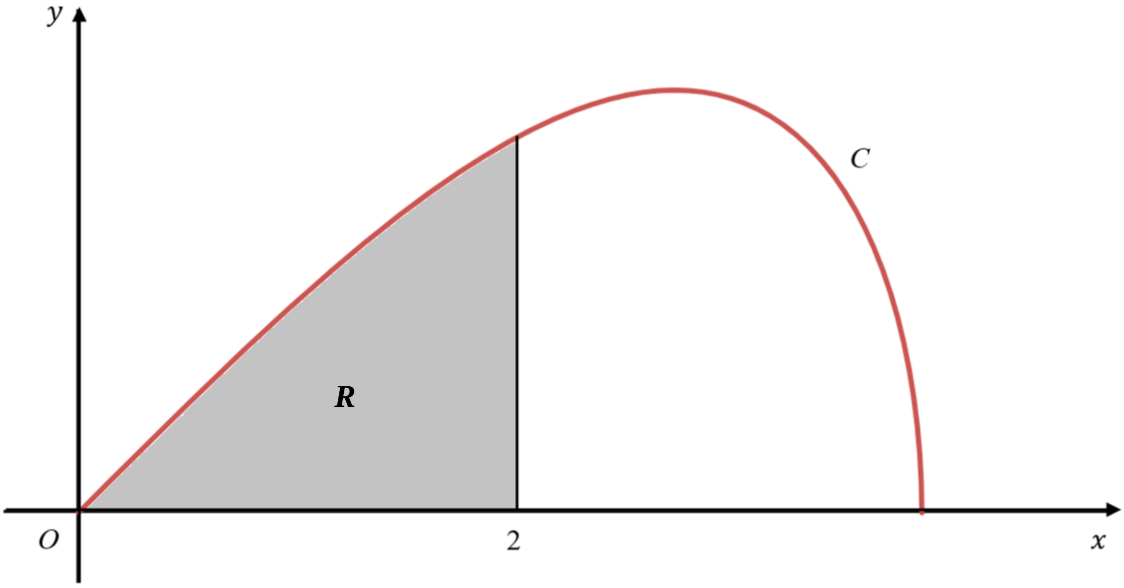


Figure 8

Figure 8 shows a sketch of the curve with parametric equations

$$x = 4\cos t \qquad y = 2\sin 2t \qquad 0 \leq t \leq \frac{\pi}{2}$$

where t is a parameter.

The finite region R is enclosed by the curve C, the x-axis and the line $x = 2$, as shown in Figure 7.

a. Show that the area of R is given by

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 16\sin^2 t\cos t \;\; \mathrm{d}t$$

(3)

b. Hence, using algebraic integration, find the exact area of R, giving in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(3)

a. M1 Attempts $y.\frac{\mathrm{d}x}{\mathrm{d}t}=(-2\sin 2t)\times 4\times\sin t$ and uses $\sin 2t = 2\sin t\cos t$.

Condone slips in finding $\frac{\mathrm{d}x}{\mathrm{d}t}$ but it must be of the form $k\sin t$

e.g. $y.\frac{\mathrm{d}x}{\mathrm{d}t}=(-2\sin 2t)\times k\sin t \ = (-4\sin t\cos t)\times k\sin t$

A1 A correct (expanded) integrand in t. Don't be concerned by the absence of

$\int$ or dt or limits

$(R)=\int -8\sin 2t \ \times\sin t \ \ \mathrm{d}t=\int -16\sin^2 t\cos t \ \ \mathrm{d}t$

A1 Correct proof with the correct limits.

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -16\sin^2 t\cos t\,\mathrm{d}t=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 16\sin^2 t\cos t \ \ \mathrm{d}t$$

where $x=0, t=\frac{\pi}{2}$ and $x=2, t=\frac{\pi}{3}$

b. M1 Attempts $\int k\sin^n t\cos t=\pm k\frac{\sin^{n+1}t}{n+1}$

e.g. $\int 16\sin^2 t\cos t=\pm 16\times\frac{\sin^3 t}{3}$

Or by substitution e.g. $u=\sin t$ to give $\pm\int ku^2 \ \ \mathrm{d}u=\pm\frac{ku^3}{3}$

A1 Any correct answer $\frac{16}{3}[\sin^3 t]_{\frac{\pi}{3}}^{\frac{\pi}{2}}=\frac{16}{3}\left[\left(\sin\frac{\pi}{2}\right)^3-\left(\sin\frac{\pi}{3}\right)^3\right]$

or appropriate limits if using substitution. $\frac{16}{3}\left[1^3-\left(\frac{\sqrt{3}}{2}\right)^3\right]$

A1 correct answer only $\frac{16}{3}-2\sqrt{3}$

(Total for Question 16 is 6 marks)